Advanced Topics and Modeling in Mathematics

Curriculum Framework

2012

Course Title: Advanced Topics and Modeling in Mathematics
Course/Unit Credit: 1
Course Number: 439050
Teacher Licensure: Please refer to the Course Code Management System (https://adedata.arkansas.gov/ccms/) for the most current licensure codes.
Grades: 9-12
Prerequisite: Algebra I, Geometry, Algebra II

Advanced Topics and Modeling in Mathematics

This course builds on Algebra I, Geometry, and Algebra II to explore mathematical topics and relationships beyond Algebra II. Emphasis will be placed on applying modeling as the process of choosing and using appropriate mathematics and statistics to analyze, to better understand, and to improve decisions in analyzing empirical situations. Collection and use of student-generated data should be an aspect of the course. Students will represent and process their reasoning and conclusions numerically, graphically, symbolically, and verbally. Students will be expected to use technology, including graphing calculators, computers, and data gathering equipment throughout the course. Teachers are responsible for including the eight Standards for Mathematical Practice found in the Common Core State Standards for Mathematics (CCSS-M). Advanced Topics and Modeling in Mathematics does not require Arkansas Department of Education approval.

Prerequisites: Algebra I, Geometry, Algebra II

Strand	Content Standard
Functions	
	1. Students will analyze and interpret functions using different representations in terms of an authentic contextual application.
	2. Students will construct and compare various types of functions and build models to represent and solve problems.
Vectors	
	3. Students will represent and model vector quantities and perform operations on vectors.
Matrix Operations	
	4. Students will perform operations on matrices and use matrices in applications.
Probability and Statistics	
	5. Students will interpret linear models, calculate expected values to solve problems, and use probability to evaluate outcomes of decisions.

An asterisk (*) indicates that the CCSS-M has identified that Student Learning Expectation as a Modeling standard

Strand: Functions

Content Standard 1: Students will analyze and interpret functions using different representations in terms of an authentic contextual application.

		Connections to CCSS-M
F.1.ATMM.1	*Interpret key features of graphs and tables in terms of two quantities for functions beyond the level of quadratic that model a relationship between the quantities	F.IF.4
F.1.ATMM.2	*Graph functions expressed symbolically and show key features of the graph using technology	F.IF.7
F.1.ATMM.3	*Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions	F.IF.7b
F.1.ATMM.4	*Graph polynomial functions, identifying *zeros* when suitable factorizations are available and showing end behavior	F.IF.7c
F.1.ATMM.5	*Graph rational functions, identifying *zeros* and asymptotes when suitable factorizations are available and showing end behavior	F.IF.7d
F.1.ATMM.6	*Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude	F.IF.7e
F.1.ATMM.7	Interpret the parameters of functions beyond the level of linear and quadratic in terms of a context	F.LE.5

Key: F.1.ATMM.1 – Functions.Content Standard 1.Advanced Topics and Modeling in Mathematics.1st Student Learning Expectation

Strand: Functions

Content Standard 2: Students will construct and compare various types of functions and build models to represent and solve problems.

		Connections to CCSS-M
F.2.ATMM.1	Model equations in two or more variables to represent relationships between quantities for functions beyond the level of linear and quadratic	A.CED.2
F.2.ATMM.2	Represent constraints or inequalities using systems of equations and/or inequalities; interpret solutions as viable or non-viable options in a modeling context for functions beyond the level of linear and quadratic	A.CED.3
F.2.ATMM.3	*Compose functions (e.g., If $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time)	F.BF.1c
F.2.ATMM.4	*Write *arithmetic* and *geometric sequences* both recursively and with an *explicit formula*; use the sequences to model situations and translate between the two forms	F.BF.2
F.2.ATMM.5	*Understand that restricting a trigonometric function to a *domain* on which it is always increasing or always decreasing allows its inverse to be constructed	F.TF.6
F.2.ATMM.6	*Use inverse functions to solve trigonometric equations that arise in modeling context; evaluate the solutions using technology and interpret them in terms of the context	F.TF.7

Key: F.2.ATMM.1 – Functions.Content Standard 2.Advanced Topics and Modeling in Mathematics.1st Student Learning Expectation

Strand: Vectors

Content Standard 3: Students will represent and model vector quantities and perform operations on vectors.

		Connections to CCSS-M
V.3.ATMM.1	Recognize *vector* quantities as having both *magnitude* and direction; represent *vector* quantities by directed line segments and use appropriate symbols for *vector* and their *magnitudes* (e.g., $\boldsymbol{v}, \lvert\boldsymbol{v}\rvert, \lvert\lvert\boldsymbol{v}\rvert\rvert, v$)	N.VM.1
V.3.ATMM.2	Find the *components of a vector* by subtracting the coordinates of an initial point from the coordinates of a terminal point	N.VM.2
V.3.ATMM.3	Solve problems involving velocity and other quantities that can be represented by *vectors*	N.VM.3
V.3.ATMM.4	Add *vectors* end-to-end, component-wise, and by the parallelogram rule; understand that the *magnitude* of a sum of two *vectors* is typically not the sum of *magnitudes*	N.VM.4a
V.3.ATMM.5	Determine the *magnitude* and direction of the sum of two given *vectors* in *magnitude* and direction form	N.VM.4b
V.3.ATMM.6	Understand *vector* subtraction; $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same *magnitude* as w pointing in the opposite direction; represent *vector* subtraction graphically by connecting the tips in the appropriate order and perform *vector* subtraction component-wise	N.VM.4c
V.3.ATMM.7	Represent scalar multiplication graphically by scaling *vectors* and possibly reversing their direction; perform scalar multiplication component-wise [e.g., as $c(vx, vy) = (c\ vx\ , c\ vy)$]	N.VM.5a
V.3.ATMM.8	Compute the *magnitude* of a scalar multiple $c\boldsymbol{v}$ using $\lvert\lvert c\boldsymbol{v}\rvert\rvert = \lvert c\rvert v$; compute the direction of $c\boldsymbol{v}$ knowing that when the $\lvert c\rvert\boldsymbol{v} \neq 0$, the direction $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ $(for\ c > 0)$ or against $\boldsymbol{v}$ $(c < 0)$	N.VM.5b

Key: V.3.ATMM.1 – Vectors.Content Standard 3.Advanced Topics and Modeling in Mathematics.1st Student Learning Expectation

Strand: Matrix Operations

Content Standard 4: Students will perform operations on matrices and use matrices in applications.

		Connections to CCSS-M
MO.4.ATMM.1	Use matrices to represent and manipulate data (e.g., to represent payoffs or incidence relationships in a network)	N.VM.6
MO.4.ATMM.2	Multiply matrices by scalars to produce new matrices (e.g., all the payoffs in a game are doubled)	N.VM.7
MO.4.ATMM.3	Add, subtract, and multiply matrices of appropriate dimensions	N.VM.8
MO.4.ATMM.4	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties	N.VM.9
MO.4.ATMM.5	Understand that *zero* and *identity matrices* play a role in matrix addition and multiplication similar to 0 and 1 in real numbers; the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse	N.VM.10
MO.4.ATMM.6	Represent a system of linear equations as a single matrix equation in a *vector* variable	A.REI.8
MO.4.ATMM.7	Find the *inverse of a matrix* if it exists, and use it to solve systems of linear equations; utilize technology to find the *inverse of matrices* with dimensions of 3 x 3 or greater	A.REI.9

Key: MO.4.ATMM.1 – Matrix Operations.Content Standard 4.Advanced Topics and Modeling in Mathematics.1st Student Learning Expectation

Strand: Probability and Statistics

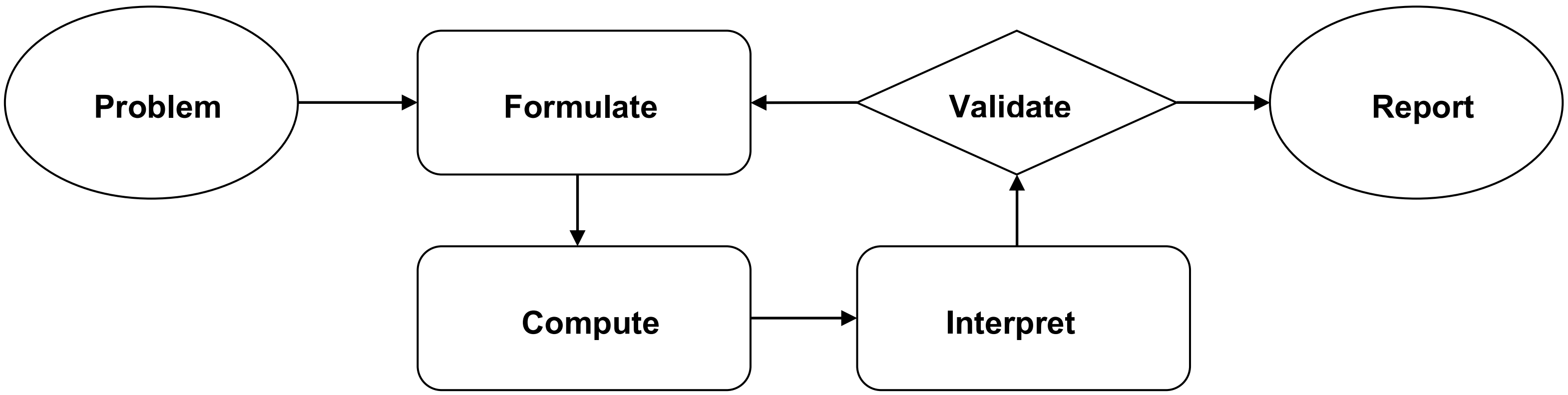
Content Standard 5: Students will interpret linear models, calculate expected values to solve problems, and use probability to evaluate outcomes of decisions.

		Connections to CCSS-M
PS.5.ATMM.1	Define a *random variable* for a quantity of interest by assigning a numerical value to each event in a *sample space*; graph the corresponding *probability distribution* using the same graphical displays as for data distributions	S.MD.1
PS.5.ATMM.2	Calculate the *expected value* of a *random variable*; interpret it as the *mean* of the *probability distribution*	S.MD.2
PS.5.ATMM.3	Develop a *probability distribution* for a *random variable* defined for a *sample space* in which *theoretical probabilities* can be calculated; find the *expected value* (e.g., find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices; find the expected grade under various grading schemes)	S.MD.3
PS.5.ATMM.4	Develop a *probability distribution* for a *random variable* defined for a *sample space* in which probabilities are assigned empirically; find the *expected value* (e.g., find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household; how many TV sets would you expect to find in 100 randomly selected households?)	S.MD.4
PS.5.ATMM.5	Find the expected payoff for a game of chance (e.g., find the expected winnings from a state lottery or a game at a fast-food restaurant)	S.MD.5a
PS.5.ATMM.6	Evaluate and compare strategies on the basis of *expected values* (e.g., compare a high-deductible versus a low-deductible automobile insurance policy using various but reasonable chances of having a minor or major accident)	S.MD.5b

Key: PS.5.ATMM.1 – Probability and Statistics.Content Standard 5.Advanced Topics and Modeling in Mathematics.1st Student Learning Expectation

Mathematical Modeling Cycle

The basic modeling cycle is summarized in this diagram. It involves: (1) identifying variables in the situation and selecting those that represent essential features; (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables; (3) analyzing and performing operations on these relationships to draw conclusions; (4) interpreting the results of the mathematics in terms of the original situation; (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable; (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



Arithmetic Sequence	A sequence in which each term after the first is equal to the previous term added to a constant value Note: constant value in an arithmetic sequence is called the common difference
Components of a Vector	Each part of a two-dimensional vector which depicts the influence of that vector in a given direction; the combined influence of the two components is equivalent to the influence of the single two-dimensional vector; the single two-dimensional vector could be replaced by the two components
Domain	The set of values of the independent variable(s) for which a function or relation is defined
Expected Value	A quantity equal to the average result of an experiment after a large number of trials
Explicit Formula	An equation in which the dependent variable can be written explicitly in terms of the independent variable
Geometric Sequence	A sequence in which each term after the first is found by multiplying the previous term by a constant, called the common ratio, *r*
Identity Matrices	A square matrix which has a 1 for each element on the main diagonal and 0 for all other elements
Inverse of a Matrix (Inverse of Matrices)	For a square matrix A, the inverse is written A^{-1}; when A is multiplied by A^{-1}, the result is the identity matrix; non-square matrices do not have inverses Note: not all square matrices have inverses; a square matrix which has an inverse is called invertible or nonsingular, and a square matrix without an inverse is called noninvertible or singular
Magnitude	The length of a vector
Mean	A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list; (e.g., for the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the mean absolute deviation is 20)
Probability Distribution	The set of possible values of a random variable with a probability assigned to each
Random Variable	An assignment of a numerical value to each outcome in a sample space
Sample Space	A list of the individual outcomes that are to be considered
Theoretical Probability	Probability is a likelihood that an event will happen $$P(event) = \frac{Number\ of\ favorable\ outcomes}{Total\ number\ of\ possible\ outcomes}$$
Vector	A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers
Zero Matrix	A matrix for which all elements are equal to 0
Zero	A value of x which makes a function $f(x)$ equal 0; a zero may be real or complex